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Maysel's Formula Generalized for Piezoelectric Vibrations: Application to Thin Shells of Revolution

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Either some or each of the layers of a composite shell made of piezoelectric materials behave as distributed actuators so that the shell becomes an intelligent or smart structure. To effectively suppress the vibrations of the shell, an electrical field with a proper control must be applied. The most efficient calculation of thermoelastic deformations is performed by Maysel's formula, i.e., within a multiple-field analysis in the isothermal background. By generalizing Maysel's formula, it becomes possible to include both the piezoelectric effects and inertia. A version is presented that allows the construction of the best auxiliary problem in the background, particularly preserving all kinds of symmetries present in the actual coupled problem. Applications of the three-dimensional formulation are illustrated for the special case of thin-layered shells of revolution, and, in addition, for a circular cylindrical shell with the piezoelectric influence function being presented. Because solutions of the auxiliary problem are more easily obtained in frequency space, the time convolution is replaced by a Fourier integral, which is eventually subjected to fast Fourier transform.

Introduction

BECAUSE of Crawley's[1] survey of the technology of intelligent or smart structures, these newly invented designs can be considered as the overlapping subset of adaptive and sensory structures. However, according to a definition given by Anderson et al.,[2] such smart structures have sensors and actuators either attached or embedded, whose actions are coordinated through a control system, which allows the structure to respond spontaneously to external stimuli. Such a concept is usually put into practice by means of integrated distributed actuators and sensors. Mostly, the distributed actuators in intelligent structures are designed by imbedding piezoelectric layers, and so, activation renders piezoelectrically induced and, as such, imposed strains, which generate the distributed input of the control system (see the recent review on piezoelectricity and its use in disturbance sensing and control of flexible structures[3]). Lee et al.[4] worked out various techniques for generating piezoelectric actuators with nonuniform spatial distributions, whereas Tzou[5] considered distributed sensing and control of piezoelectric shells.

This paper is concerned, in detail, with actuating piezoelectric effects in (thin) shells of revolution, where special emphasis is given to the identification of the piezoelectric actuation as a source of self-stress. It is demonstrated that piezoelectrically induced strains, within a multiple-field approach, can be interpreted conveniently as eigenstrains acting in the background shell (cf. Irschik et al.[6] for the simpler case of the vibrations of beams). Such an interpretation is especially important for applications because efficient solution strategies exist in the related field of thermoelasticity (see the presentation of important topics in the field of thermal stresses[7]). Elastic thermal stresses may serve as an illustration of the special mechanisms that are connected with eigenstrains in general multiple-field analyses with a properly defined (isothermal) background. Occasionally, thermal stresses and surface traction at clamped boundaries may occur without any deformations, and, similarly, there may be thermal strains without stresses. Thermal stresses satisfy the equations of dynamic equilibrium with zero body forces, and the eigenstrains enter the constitutive relations in an explicit manner. The identification and interpretation of piezoelectrically induced strains as eigenstrains constitute the key for the understanding of piezoelectric actuation. Indeed, it generally is assumed that the analogy between the piezoelectric effect and the thermal and hygrothermal effect is extremely important because it enables the analyst and designer to utilize all available thermoelastic and hygrothermal solutions to solve problems involving piezoelectric materials.[8]

By means of such an eigenstrain analysis, a convolution integral representation is achieved for transient piezoelectrically induced vibrations of elastic bodies and is then applied to transient vibrations of (thin) shells of revolution. Efficiency of the multiple-field analysis is enhanced by considering an auxiliary problem, namely the forced vibrations of the background structure attributable to imposed forces, yet without piezoelectric effects. The latter auxiliary problem refers to the conventional solution of (shell) dynamics. The convolution integral, as such, links the piezoelectric vibrations to the dummy vibrations of the auxiliary problem, and thus represents a generalized Maysel's formula, which originally was developed for thermoelasticity (see the literature on statics of thermally loaded thin shells of revolution,[9] the state of the art of statics,[10] and a three-dimensional dynamic eigenstrain formulation[11,12]). Subsequently, this latter formulation of the multiple-field approach is extended to the transient vibrations of thin multilayered shells of revolution, taking into account the piezoelectric effects. Furthermore, for a circular cylindrical shell, the solution is given in terms of a Fourier integral that is numerically evaluated by fast Fourier transform (FFT).

Convolution Integral for Piezoelectrically Induced Vibrations of Elastic Bodies

The vibrations of a linear elastic, inhomogeneous, and anisotropic body exhibiting linear piezoelectric effects are considered in the context of a multiple-field analysis. It is assumed that a stable equilibrium configuration exists and the application of the geometrically linearized theory of continuum mechanics is justified. Piezoelectricity is utilized to induce vibrations of the body via the inverse piezoelectric effect.[3] It is within the scope of this general section to derive a problem-oriented convolution integral for piezoelectrically induced transient vibrations with homogeneous initial conditions assigned. A Cartesian coordinate system, x_i, $i = 1, 2, 3$, is used, and Einstein's summation convention about repeated indices

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is understood. The generalized Hooke's law linearly relates stress and strain:

$$\sigma_{ij} = c_{ijkl}\left(\varepsilon_{kl} - \varepsilon^*_{kl}\right); \qquad i, j, k, l = 1, 2, 3 \tag{1}$$

where the conditions of symmetry restrict the fourth-order elasticity tensor

$$c_{ijkl} = c_{jikl} = c_{ijlk} = c_{klij} \tag{2}$$

In Eq. (1), ε^*_{kl} denotes a field of sources of self-stress applied to the linear elastic background.[13] Their interpretation as the imposed strains allows for a multiple-field analysis and also relates the formulation to that of the continuum theory of dislocations.[14] In the case of piezoelectrically induced vibrations, the strains are proportional to the components E_m of the vector of the electric-field strength

$$\varepsilon^*_{kl} = d_{mkl} E_m \tag{3}$$

and the piezoelectric constants are subjected to the conditions of symmetry[3,5,15]:

$$d_{mkl} = d_{mlk} \tag{4}$$

Recall the coupling between the electric-field strength and the mechanical strain as expressed by the direct piezoelectric effect. However, in practical applications, the latter effect frequently may be neglected, depending on the amount of field strength necessary for inducing mechanical vibrations. Nevertheless, the following formulas remain valid for the coupled case, too.

A general reciprocity principle that is valid for the vibrations of anisotropic, thermoelastic, piezoelectric bodies has been derived[15] in the frequency domain. Subsequently, isothermal conditions are considered for the sake of brevity, but thermal strains can be incorporated easily in the sources of self-stress ε^*_{kl}. In Nowacki's formulation,[15] two states of loading of the background body are considered [cf. Eq. (5.8.28) of Ref. 15], keeping the boundary conditions in both cases unchanged. These two states are denoted by attaching the superscripts (a) and (b). The inverse transformation, from the frequency domain to the time domain, yields, with homogeneous initial conditions at $t = 0$ taken into account,

$$\begin{aligned}
&\int_0^t \int_V \left[f_i^{(a)}(\boldsymbol{\xi}; \tau) u_i^{(b)}(\boldsymbol{\xi}; t-\tau)\right.\\
&\quad \left. - f_i^{(b)}(\boldsymbol{\xi}; t-\tau) u_i^{(a)}(\boldsymbol{\xi}; \tau)\right] \mathrm{d}V(\boldsymbol{\xi})\,\mathrm{d}\tau\\
&\quad + \int_0^t \int_V \left[c_{ijkl}\varepsilon_{kl}^{*(a)}(\boldsymbol{\xi}; \tau)\varepsilon_{ij}^{(b)}(\boldsymbol{\xi}; t-\tau)\right.\\
&\quad \left. - c_{ijkl}\varepsilon_{kl}^{*(b)}(\boldsymbol{\xi}; \tau)\varepsilon_{ij}^{(a)}(\boldsymbol{\xi}; t-\tau)\right] \mathrm{d}V(\boldsymbol{\xi})\,\mathrm{d}\tau = 0
\end{aligned} \tag{5}$$

The volume of the body is denoted by V, and the vector $\boldsymbol{\xi}$ samples the spatial variables of integration. Time is denoted by t, and τ is the time variable of integration. Densities of (external) body forces are characterized by the components f_i, and u_i are the mechanical displacements in the two states with superscripts (a) and (b) attached.

Equation (5) must be specified in a problem-oriented analysis: Hence, state (a) is identified with the piezoelectrically induced vibrations without imposed body forces,

$$f_i^{(a)} = 0 \tag{6}$$

Furthermore, case (b) is taken as the auxiliary dummy force problem of the background that thus is kept free of any imposed strains:

$$\varepsilon_{kl}^{*(b)} = 0 \tag{7}$$

Taking into account the conditions of symmetry, Eq. (2), and reducing Hooke's law of Eq. (1) for the auxiliary problem by nullifying the imposed strains,

$$\sigma_{kl}^{(b)} = c_{klij}\varepsilon_{ij}^{(b)} \tag{8}$$

transforms the kernel of the second integral in Eq. (5) to

$$c_{ijkl}\varepsilon_{kl}^{*(a)}\varepsilon_{ij}^{(b)} = c_{klij}\varepsilon_{ij}^{(b)}\varepsilon_{kl}^{*(a)} = \sigma_{kl}^{(b)}\varepsilon_{kl}^{*(a)} \tag{9}$$

Substituting Eqs. (6–9) into Eq. (5) incorporates the auxiliary problem and finally gives

$$\begin{aligned}
&\int_0^t \int_V f_i^{(b)}(\boldsymbol{\xi}; t-\tau) u_i^{(a)}(\boldsymbol{\xi}; \tau)\,\mathrm{d}V(\boldsymbol{\xi})\,\mathrm{d}\tau\\
&\quad = \int_0^t \int_V \varepsilon_{kl}^{*(a)}(\boldsymbol{\xi}; \tau)\sigma_{kl}^{(b)}(\boldsymbol{\xi}; t-\tau)\,\mathrm{d}V(\boldsymbol{\xi})\,\mathrm{d}\tau
\end{aligned} \tag{10}$$

This convolution integral formulation represents the dynamic extension of Maysel's formula originally developed for quasistatic thermoelasticity[10] to the more general case of piezoelectrically induced strains. Equation (10) applies even to the nonlinear dynamic theory of thermoelastoviscoplasticity: In that case, the inelastic parts of strain are interpreted as the imposed strains acting in the linear elastic background material. In this context, the above derivation has been discussed in more detail elsewhere.[12] For an alternative derivation, see Irschik et al.[11]

The integral on the left-hand side of Eq. (10) is further transformed to make its application more comfortable. For this sake, the dummy body forces in the auxiliary problem are instantaneously applied at time τ, with the observation instant at time $t \geq \tau$:

$$f_i^{(b)}(\boldsymbol{\xi}; t-\tau) = F_i^{(b)}(\boldsymbol{\xi})\delta(t-\tau) \tag{11}$$

and the properties of Dirac's delta function δ allow the timewise integration in the closed interval $0 \leq \tau \leq t$:

$$\begin{aligned}
&\int_0^t \int_V f_i^{(b)}(\boldsymbol{\xi}; t-\tau) u_i^{(a)}(\boldsymbol{\xi}; \tau)\,\mathrm{d}V(\boldsymbol{\xi})\,\mathrm{d}\tau\\
&\quad = \int_V F_i^{(b)}(\boldsymbol{\xi}) u_i^{(a)}(\boldsymbol{\xi}; t)\,\mathrm{d}V(\boldsymbol{\xi})
\end{aligned} \tag{12}$$

Note that the remaining volume integral on the right-hand side of Eq. (12) may be interpreted as the total virtual work $W^{(b,a)}(t)$ of the spatial dummy force distribution $F_i^{(b)}(\boldsymbol{\xi})$ done on the piezoelectrically induced deformations at time t, $u_i^{(a)}(\boldsymbol{\xi}; t)$,

$$W^{(b,a)}(t) = \int_V F_i^{(b)}(\boldsymbol{\xi}) u_i^{(a)}(\boldsymbol{\xi}; t)\,\mathrm{d}V(\boldsymbol{\xi}) \tag{13}$$

Thus, Eq. (10) takes on the form required for straightforward structural applications:

$$W^{(b,a)}(t) = \int_0^t \int_V \varepsilon_{kl}^{*(a)}(\boldsymbol{\xi}; \tau)\sigma_{kl}^{(b)}(\boldsymbol{\xi}; t-\tau)\,\mathrm{d}V(\boldsymbol{\xi})\,\mathrm{d}\tau \tag{14}$$

The kernel functions $\sigma_{kl}^{(b)}(\boldsymbol{\xi}; t-\tau)$ denote the dynamic stresses at time t attributable to the impulsive dummy body force distributions $F_i^{(b)}$ applied at time τ in the interval $0 \leq \tau \leq t$. Equation (14), still of general validity, is specified subsequently for piezoelectrically induced axisymmetric vibrations of (thin) shells of revolution. The result is an extension to dynamics and to piezoelectricity of the analysis of quasistatic thermal stresses in thin shells of revolution of Scheidl and Ziegler.[9]

Piezoelectrically Induced Vibrations of Layered Shells of Revolution

Thin shells, composed of transversely isotropic and perfectly bonded layers, are considered under axisymmetric conditions. Either some or each of the layers are made of piezoelectric materials. In spite of the layered structure of the shell wall, approximate shell theories are applied that keep the in-plane dummy normal stresses (in the auxiliary problem) linearly distributed over the thickness of the layers. Taking the coordinate x_3 to be perpendicular to the shell surface and (x_1, x_2) to be principal coordinates, i.e., pointing in the direction of the principal curvatures of the shell, the in-plane normal stresses of the auxiliary problem are linearly related to the overall stress resultants in a simple manner if a constant Poisson's ratio is assumed to hold across the layers and curvature effects are negligible:

$$\sigma_{11}^{(b)} = k\left[\frac{n_1^{(b)}}{D} + \frac{m_1^{(b)}}{B}x_3\right], \qquad \sigma_{22}^{(b)} = k\left[\frac{n_2^{(b)}}{D} + \frac{m_2^{(b)}}{B}x_3\right] \tag{15}$$

Normal forces and bending moments per unit of length are denoted $n^{(b)}$ and $m^{(b)}$, respectively. The factor k varies from layer to layer and is proportional to the local modulus of elasticity, and the effective stiffnesses are denoted by D and B. For instance, in the case of the classic laminate theory of composite shells that are made of isotropic layers in a symmetrical arrangement, and with the assumptions made above, the parameters take on the simplified and approximated form[16,17]

$$k = \frac{E}{1-\upsilon^2}, \qquad D = (1-\upsilon^2)^{-1}\int_{-h/2}^{h/2} E(x_3)\,\mathrm{d}x_3$$
$$B = (1-\upsilon^2)^{-1}\int_{-h/2}^{h/2} x_3^2 E(x_3)\,\mathrm{d}x_3 \tag{16}$$

Furthermore, it is assumed that the piezoelectric continuum is subjected to a transverse electric field E_3 that is perpendicular to the surface of the shell, and hence, the remaining components

$$E_1 = E_2 = 0 \tag{17}$$

vanish. Proper dielectric constants are[5]

$$d_{311} = d_{322} = d_3, \qquad d_{312} = d_{321} = 0 \tag{18}$$

Integration over the wall thickness h is performed and the surface integral remains on the right-hand side of Eq. (14):

$$\int_V \varepsilon_{kl}^{*(a)}\sigma_{kl}^{(b)}\,\mathrm{d}V = \int_S \left(n^{(b)}\varepsilon^E + m^{(b)}\kappa^E\right)\mathrm{d}S \tag{19}$$

where superscript (b) is understood, and

$$n = n_1 + n_2, \qquad m = m_1 + m_2 \tag{20}$$

denote the first invariant of the normal force and of the bending moment tensor, respectively. The mean of the induced piezoelectric strain in Eq. (19) under conditions of transverse isotropy is

$$\varepsilon^E = \frac{1}{D}\int_h k d_3 E_3\,\mathrm{d}x_3 \tag{21}$$

and the piezoelectrically induced mean curvature is defined by the thickness moment of first order

$$\kappa^E = \frac{1}{B}\int_h x_3 k d_3 E_3\,\mathrm{d}x_3 \tag{22}$$

The complete analogy between the piezoelectrically induced strains and thermal strains is recognized by inspection of Eqs. (19–21) (for definitions in the latter case, see Ziegler and Irschik[10]).

Axisymmetric Vibrations

The transient vibrations of shells of revolution excited by axisymmetric distributions of piezoelectrically induced strains are considered next. The geometry of the shell is shown in Fig. 1. A unit ring line load, applied at the latitude angle ϕ, is selected as the dummy force distribution, to keep the auxiliary problem axisymmetric (see Scheidl and Ziegler[9] and Ziegler[18] for the case of quasistatic axisymmetric thermal stresses).

Lateral unit ring forces are considered at first. In that case the virtual work in Eq. (13) becomes

$$W^{(b,a)}(t) = 2\pi r w(\phi; t) \tag{23}$$

where the piezoelectrically induced deflection is denoted by $w(\phi; t)$. Those lateral ring forces acting at the parallel of latitude, $\phi =$ const, produce membrane forces and moments. When summed, the amounts that are assumed to be known are denoted $n_{(n)}^{(b)}(\phi^*, \phi)$ and $m_{(n)}^{(b)}(\phi^*, \phi)$, respectively, where the latitude in the observation points is ϕ^*. The subscript (n) refers to the radial (normal) direction of the dummy force. Equation (14) finally becomes an integral over the latitude angle, r_1^* is the radius of curvature of the meridian (see Fig. 1):

$$2\pi r w(\phi; t) = \int_0^t\int_{\phi_1}^{\phi_2}\left[n_{(n)}^{(b)}(\phi^*, \phi; t-\tau)\varepsilon^E(\phi^*; \tau)\right.$$
$$\left. + m_{(n)}^{(b)}(\phi^*, \phi; t-\tau)\kappa^E(\phi^*; \tau)\right]2\pi r^* r_1^*\,\mathrm{d}\phi^*\,\mathrm{d}\tau \tag{24}$$

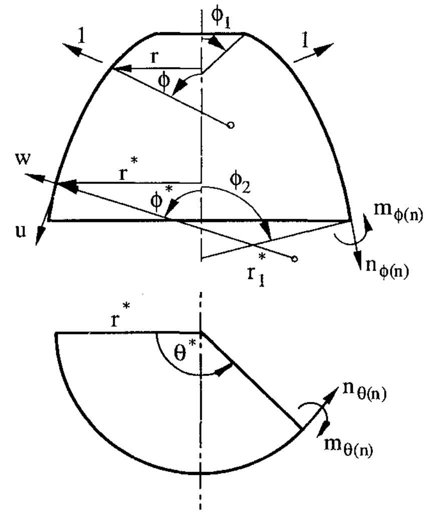

Fig. 1 Thin shell of revolution: auxiliary problem with lateral unit ring load applied.

Analogously, the piezoelectrically induced tangential displacement results from applying a tangential unit ring load in the auxiliary problem; the index (t) refers to this direction:

$$2\pi r u(\phi; t) = \int_0^t\int_{\phi_1}^{\phi_2}\left[n_{(t)}^{(b)}(\phi^*, \phi; t-\tau)\varepsilon^E(\phi^*; \tau)\right.$$
$$\left. + m_{(t)}^{(b)}(\phi^*, \phi; t-\tau)\kappa^E(\phi^*; \tau)\right]2\pi r^* r_1^*\,\mathrm{d}\phi^*\,\mathrm{d}\tau \tag{25}$$

Concentrated Piezoelectric Sources in Circular Cylindrical Shells

The deflection of a circular cylindrical shell of radius $r = r^* =$ const is included in Eq. (24), which in that case reduces to an integral over the length L of the shell, and x points in the axis direction:

$$w(x; t) = \int_0^t\int_L \left[n^{(b)}(\xi, x; t-\tau)\varepsilon^E(\xi; \tau)\right.$$
$$\left. + m^{(b)}(\xi, x; t-\tau)\kappa^E(\xi; \tau)\right]\mathrm{d}\xi\,\mathrm{d}\tau \tag{26}$$

The subscript (n) is omitted. The axial locations are denoted by x and ξ, and the dummy ring load is applied at x. Equation (26) is used to derive piezoelectric dynamic influence functions. Consider the case of a unit concentrated impulsive piezoelectric ring source of the curvature type that is applied at the axial location ρ at time $\gamma \leq t$:

$$\kappa^E(\xi; \tau) = \delta(\xi - \rho)\delta(\tau - \gamma), \qquad \varepsilon^E(\xi; \tau) = 0 \tag{27}$$

Substituting this product of Dirac's delta functions in Eq. (26) yields by spatial and timewise integration

$$w(x, \rho; t - \gamma) = m^{(b)}(\rho, x; t - \gamma) \tag{28}$$

The deflection in x attributable to a concentrated impulsive piezoelectric ring source acting in ρ is thus related by convolution to the sum of moments in ρ of the auxiliary problem with an impulsive concentrated unit ring load applied at x. The sum of moments in Eq. (28) may be taken from the vast literature on circular shells. Note that high concentrations of deformation have to be expected in the neighborhood of the concentrated source (cf. Lukasiewicz[19] for static stresses in locally loaded shells).

In practice, however, the evaluation of Eqs. (26) and (28) is conveniently performed in the frequency space. For that sake, we consider the Fourier transforms $f(\omega)$, $g(\omega)$ of two time-dependent functions $F(t)$ and $G(t)$. The convolution theorem states that the inverse Fourier transform of the product $f(\omega)g(\omega)$ is the convoluted integral of the time functions,

$$\frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} G(\tau)F(t-\tau)\,\mathrm{d}\tau$$

According to that theorem, Eq. (26) is transformed in the frequency space:

$$w(x;\omega) = \int_L \left[n^{(b)}(\xi, x;\omega)\varepsilon^E(\xi;\omega) + m^{(b)}(\xi, x;\omega)\kappa^E(\xi;\omega)\right] d\xi \tag{29}$$

where the functions defined in time space and frequency space are to be distinguished. Note that the limits of time integration in Eq. (26) may be extended to infinity, because the sources do vanish for $\tau < 0$, and causality applies. In particular, Eq. (28) becomes

$$w(x,\rho;\omega) = m^{(b)}(\rho, x;\omega) \tag{30}$$

The radial deformation of the layered cylindrical shell in the auxiliary problem is readily available in the frequency space where an analogy to the deflection of a beam on a Winkler foundation holds.[8] For the infinitely long shell,[20]

$$w^{(b)} = \frac{\exp(-\beta|\rho - x|)}{8B\beta^3}(\cos\beta|\rho - x| + \sin\beta|\rho - x|) \tag{31}$$

where

$$\beta^4 = \frac{(1-\nu^2)D/r^2 - \mu\omega^2}{4B} \tag{32}$$

and μ denotes the mass density per unit of area.

Hence, by differentiating twice, the sum of moments in the auxiliary problem becomes

$$m^{(b)} = \frac{\exp -\beta|\rho - x|}{4\beta}(\cos\beta|\rho - x| - \sin\beta|\rho - x|) \tag{33}$$

Using the shift property of the Fourier transformation and subsequently applying the FFT renders the result in the time domain of Eq. (28).

Because static solutions for beams on a Winkler foundation are more readily available and substitution of the dynamic stiffness for the Winkler parameter yields the proper frequency response function, the impulse response of the auxiliary problem in Eq. (26) in general, should be replaced by its inverse Fourier integral representation, Eq. (29). Thus, a final result is achieved by a spatial integration with the possibility of application of FFT apparent:

$$w(x;t) = \frac{1}{\sqrt{2\pi}}\int_{-\infty}^{\infty} e^{-i\omega t} w(x;\omega)\, d\omega \tag{34}$$

Conclusions

It is demonstrated that the identification and interpretation of piezoelectrically induced strains as eigenstrains constitute the key for the understanding of piezoelectric actuation. A multiple-field analysis is thus presented that results in a convolution integral representation for transient piezoelectrically induced vibrations of elastic bodies with a further specialization to smart (thin and multilayered) shells of revolution. Efficiency of the analysis is enhanced by considering an auxiliary problem, namely the forced vibrations of the background structure attributable to imposed forces, yet without piezoelectric effects. The latter auxiliary problem then refers to the conventional solution of (shell) dynamics. The convolution integral, as such, links the piezoelectric vibrations to the forced vibrations of the background, and thus represents a generalized Maysel's formula, which originally was developed for thermoelasticity. Concentrated piezoelectric sources in circular cylindrical shells are considered to demonstrate the efficiency and versatility of the method. Because the classical solutions of the auxiliary problem, i.e., of the vibrations of the background structure, are more readily available in frequency space, we recommend application of FFT in the course of evaluations. For the cylindrical shell, even the static solution of a beam on a Winkler foundation suffices to derive the frequency response function, which then enters the Fourier integral.

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